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RESEARCH ARTICLE

THE IMPACT OF THE DENSITY OF STAR R&D NETWORKS ON THE MAXIMUM OUTCOMES

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ABSTRACT

While the empirical findings have confirmed the importance of the centers in preserving the characteristic features of the R&D network, this paper will focus theoretically on central firms in the star networks. The analysis of the equilibrium outcomes reveals the impact of the market structure on centralizing of firms and on expanding the star network. It seems that when the differentiation rate between products of firms is high, the development of the star network contributes to increasing the individual and social outcomes. Whereas, if the substitution between the products is high, the benefit behind growing the star network limits to firms that form new R&D partnerships and to the total welfare.

INTRODUCTION

There has been substantial progress in network research reported in both the theoretical and empirical literature. A network is formed by a set of vertices (nodes) and a set of edges (links) connecting these vertices (Newman, 2003; Jackson, 2008). In the real world networks of business, there are massive examples, including diffusion of the knowledge among firms, job-contacts, sellers and buyers, and R&D cooperation among firms or between firms and institutions. The focus of this paper is on the cooperation among firms in R&D where R&D partnerships are described as a network (called an R&D network). In a such network, the players (firms) are represented by nodes and the R&D partnerships (cooperation) are represented by links (Goyal and Moraga-Gonzalez, 2001; Cantner and Graf, 2006; Deroian, 2008;Westbrock, 2010; Konig *et al*., 2012).

In the empirical R&D literature, several researchers investigated the patterns through which R&D cooperation networks of worldwide firms develop and grow. Most of these studies found that the cooperation exhibit characteristic features of complex networks that describe many of the social networks (Ahuja, 2000; Stuart, 2000; Verspagen and Duysters, 2004; Tomasello *et al*., 2013). The authors have found that highly connected firms (firms with many links) have a role in constructing such complexity. Stuart (2000) and Ahuja (2000) explored the highly connected firms in the continuing development of the small world network.[1] They showed that most of the existing and new entering firms seek to link to the highly connected firms. Tomasello *et al*. (2013) confirmed this result where they found that the cores of the network exist and dominate the network structure.

Another important issue that has been investigated in the R&D networks is the distribution of the cooperative links between firms (degree distribution). The common finding in many papers (e.g., Riccaboni and Pammolli, 2002; Powell *et al*., 2005) is that the construction and development of a small world network are based on the existence of the highly connected firms (cores). From this, if firms' position in the network is a matter for the performance, one would suspect that the highly connected firms who dominate the network would not exit the system of the collaboration.

In the theoretical R&D literature, there are many papers explored the establishment of the star networks. Goyal and Moraga-Gonzalez (2001) in the class of asymmetric networks stated that the center of the star network obtains a high profit, but the overall network is not socially preferable. Goyal and Joshi (2003) found that the network consisted of interlinked stars is the stable structure that ensures high profits for all firms. Song and Vannetelbosch (2007) considered Goyal and Moraga-Gonzalez model for firms belong to different countries. They

[1]A small world network can be defined as a graph in which most nodes are not linked to one another, but they can be reached from every other node by only a small number of links Watts and Strogatz (1998).

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studied the case when there are three firms producing homogeneous goods for asymmetric R&D networks. Their study focused on how government subsidies to R&D affect the stability and efficiency of international R&D collaboration networks. They found that in most cases with the subsidies, the star network is the strongly efficient network.[2]

In this paper, we use the theoretical R&D model by Goyal and Moraga-Gonzalez (2001) to studystar R&D networks.[3] The star network defined as a node located at the center of the network and linked to other nodes (periphery) where the latter nodes are not linked to each other. Goyal and Moraga-Gonzalez studied asymmetric networks, but they did not focus on the star network. The contribution of this paper lies in studying the impact of increasing centralization in the network on the equilibrium outcomes. Also, the paper concerns the change of the density of the star network resulting from increasing the size of the periphery or the connection between them.[4]

In particular, we try to answer the following three related questions:

1. How does the size of the R&D cooperation within the star network influence the outcomes of the central firm?
2. Does the increase of the periphery size carry positive impacts on the aggregate outcomes?
3. How does the market structure impact the star network growth?

We answer the previous questions by considering a finite population of firms with a linear quadratic utility function under Cournot competition for two cases of the market structure: independent and homogeneous goods.[5]

In this paper, we find that a large network of periphery always generates high profits for the central firms, regardless of the market structure. Nevertheless, the maximum profit of the central firm with homogeneous goods depends on the lack of the cooperation between firms in the periphery, meaning that as the cooperative links among firms in the periphery decrease, the central firm has a higher profit. In contrast, with varying the products, the optimal profit of the center increases with increasing the cooperation between firms in the periphery. These results consist with the empirical findings that indicate to the presence of the dominant group of central firms characterized the structure of the R&D network. Such firms have a role in preserving the network frame by attracting many new firms into their R&D system which in turn contributes to improving the knowledge rate of the centers.

The cooperation advantages of the new firms that settle in the periphery of the center sometimes carry disadvantages to its competitors. If the rate of the substitution between the products is high, the individual outcomes of the existing peripheral firms are negatively affected by growing the periphery size. If the products are differentiated, contrary to the previous case, the cooperation advantages of the new peripherals involve the existing firms. When examining the impact of the new peripheral firms on the aggregate outcomes, it seems that the aggregate quantity and the total welfare are better, irrespective of the product type.

Finally, the connectivity of the star networks sometimes generates high outcomes. The network that is consisting of interlinked stars is profitable and more efficient if the products are differentiated. However, if they are very substitutes, the connectivity of the stars generates high profit for the centers, but low R&D efforts and productions for all firms. Also, the interlinked stars seem less efficient than the disconnected stars. This indicates that the social incentive for interlinking the centers may be weak if the substitution between the products is high.

This paper is organized as follows. In the second section, we review issues of the social network and we introduce the Goyal and Moraga-Gonzalez (2001) model. Then, we review some economic terminologies. In the third section, we study the star networks under Goyal and Moraga-Gonzalez model. In the fourth section, we conclude our study.

BACKGROUND

Network

A network is an ordered pair $G(N, E)$ where $N = \{i, j, k, ...\}$ is a set of nodes connected by links $E = \{ij, jk, ...\}$ (Newman, 2003; Jackson, 2008). We consider undirected networks in the sense that each link between any two nodes runs in both directions. We also consider simple networks that have no parallel links (links that have the same end nodes) or loops (links where their start and end nodes are the same).

A sequence of links between any two nodes is called a path. Therefore, two nodes are connected if there is a path between them; otherwise they are disconnected. The set of nodes that are linked to node i is defined as a neighbor set of node i: $Ni = \{j \in N : ij \in E\}$. The length of the neighbors' set of node i is a degree of that node (i.e., $deg(i)=|Ni|$). If $|N| = n$ and $|E| = m$, the density of the network is $D = 2m/n(n-1)$.

A subgraph $G'(N', E')$ of the network $G(N, E)$ is a graph such that $N' \subseteq N$ and $E' \subseteq E$. A component of a network is a nonempty sub-graph in which any two vertices are connected to each other by paths. An example of components is a star network. Such network consists of a center node located at the center of the network and linked to other nodes (periphery) that are not linked to each other. The interlinked stars mean star networks linked by the centers or by some firms that are in the peripheries.

The model used in this paper depends on the network concept and on the economic model. The firms' aim in conducting R&D is to minimize the cost of the production to maximize the profit of firms.

R&D Network Model

In the R&D network, nodes represent firms and links represent

[2]There are many other recent papers on R&D network, which also discussed the star networks (e.g., Galeotti and Melndez, 2004; Galeotti *et al*., 2006).

[3]Goyal and Moraga-Gonzalez (2001) described research and development (R&D) cooperation between firms as a network game. The model has three stages: network formation, R&D investment and market competition.
The effective effort of each firm equals the own effort and efforts of other firms that are determined by a free R&D spillover. If firms are linked, the spillover between them is set one; otherwise it is set free less than one.

[4]When we say size of peripheral firms, we mean the number of firms in the periphery.

[5]For complementary (substituted) goods, we expect that the outcomes are similar to the case when the products are independents (homogeneous).

R&D partnerships.[6] Since the R&D cooperation is a mutual benefit, each link between any two firms runs in both directions (i.e., undirected networks).

We consider the R&D network model by Goyal and Moraga-Gonzalez (2001) where the cooperation is modeled as a three-stage game. In the first stage, firms choose their partners and the cooperating firms are joining together via links to form a network. The R&D spillover occurs between any two non-cooperating firms. In the second stage, firms choose their level of cost reducing R&D effort. In the third stage, firms compete by setting their products (Cournot competition). In Goyal and Moraga-Gonzalez, the effective R&D effort for each firm is described by the following equation:

$$X_i = x_i + \sum_{j\in N_i} \boldsymbol{x_j} + \beta \sum_{k\notin N_i} \boldsymbol{x_k}, \quad \boldsymbol{i = 1, \dots, n}, \tag{1}$$

Where $x_i>0$ denotes R&D investment of firm i, Ni is the set of firms participating in R&D with firm i and $\beta\in[0,1)$ is the R&D spillover. The effective R&D investment reduces firm i's marginal cost of production:

$$c_i = \bar{c} - x_i - \sum_{j\in N_i} \boldsymbol{x_j} - \beta \sum_{k\notin N_i} \boldsymbol{x_k}, \quad \boldsymbol{i = 1, \dots, n}, \tag{2}$$

Where $\bar{c}$ is the marginal cost of the production.

Economic Model

We consider the inverse demand function given in the following equation:

$$D_i^{-1} = \boldsymbol{p_i = a - q_i - \lambda \sum_{j\neq i} q_j}, \quad \boldsymbol{i = 1, \dots, n}, \tag{3}$$

Where $a>0$ denotes the willingness of consumers to pay and the parameters pi and qi are the price and quantity of good i, respectively. The parameter $\lambda\in[-1,1]$ is the differentiation degree where if $\lambda<0$ ($\lambda>0$), goods are complements (substitutes). In this paper, we consider the case when the goods are independent (λ=0) and homogeneous (λ=1).

The effort is assumed to be costly and the function of the cost is quadratic, so that the cost of R&D is γx_i^2, where $\gamma>0$ (D'Aspremont and Jacquemin, 1988). The profit π_i for firm i is the difference between revenue and production cost minus the cost of R&D

$$\pi_i = (p_i - c_i)q_i = (a - q_i - \lambda \sum_{j\neq i}^{n} q_j - c_i)q_i - \gamma x_i^2 \tag{4}$$

Where c_i is the production cost given by equation 2.
The total Welfare (TW) is the total surplus of consumers plus the industry profit

$$TW = \frac{(1-\lambda)}{2}\sum_{i=1}^{n} q_i^2 + \frac{\lambda}{2}\left(\sum_{i=1}^{n} q_i\right)^2 + \sum_{i=1}^{n} \pi_i \tag{5}$$

For the equilibrium, we assume that the marginal cost $\bar{c}$ is constant and equal for all firms. We identify the subgame perfect Nash equilibrium by using backwards induction.

[6]Practically, the network G is represented by an $n \times n$ adjacency symmetric matrix A with elements 0 or 1, depending on whether or not firms are linked. Thus, if aij= 1, firms i and j are linked (firms i and j cooperate in R&D), and aij= 0 otherwise.

Under Cournot competition, we solve for any firm i $\frac{\partial \pi}{\partial q_i} = 0$. This yields the best response function of quantity of good i:

$$\mathbf{q_i} = \frac{\mathbf{a - c_i - \lambda \sum_{j\neq i} q_j}}{\mathbf{2}}. \tag{6}$$

Substituting the best response functions (equation 6 for each i) into each other yields the symmetric equilibrium that is Nash equilibrium for the production quantity:

$$q_i = \frac{(2-\lambda)a - (2+(n-2)\lambda)c_i + \lambda\sum_{j\neq i} c_j}{(2-\lambda)((n-1)\lambda+2)} \tag{7}$$

By substituting (7) into the profit function (4), the equilibrium profit is

$$\pi_i = \left[\frac{(2-\lambda)a - (2+(n-2)\lambda)c_i + \lambda\sum_{j\neq i} c_j}{(2-\lambda)((n-1)\lambda+2)}\right]^2 - \gamma x_i^2. \tag{8}$$

Calculating the equilibrium investment x_i depends on the structure of the R&D network. By knowing the structure, we find the cost function c_i to substitute it into the profit function (8). Then, we calculate the best response function of R&D investment for each firm i $(\partial^2\pi/\partial x^2 > 0)$ By plugging them into each other, we have the symmetric equilibrium for the R&D investment.

Note that, the parameter $\gamma > 0$ should be high to avoid negative outcomes. To have suitable values of γ, the effort and cost functions should be non-negative and the second order condition for maximizing profit function$(\partial^2\pi/\partial x^2 > 0)$ should be satisfied. According to Goyal and Moraga-Gonzalez, we have

$$\begin{cases} \gamma > \max\left\{\frac{an}{4\bar{c}}, \frac{n}{4}\right\} \; if\ \lambda = 0 \\ \gamma > \max\left\{\frac{a}{4\bar{c}}, \frac{n^2}{(n+1)^2}\right\} \; if\ \lambda = 1 \end{cases} \tag{9}$$

Henceforth, $\gamma_{\lambda_0}^*$ and $\gamma_{\lambda_1}^*$ are suitable values for the equilibrium outcomes under independent and homogeneous goods, respectively.

The study of R&D cooperation under the network game involves the concepts of pairwise stability and efficiency. The pairwise stability depends on firms' profit functions and it is a necessary condition for strategic stability as shown in (Jackson and Wolinsky, 1996).

Definition 1 (Pairwise Stability) *For any network G to be stable, the following two conditions need to be satisfied for any two firms i, j ∈ G:*

1. *If ij ∈ G, $\pi_i(G) \geq \pi_i(G - ij)$ and $\pi_j(G) \geq \pi_j(G - ij)$,*
2. *If ij G and if $\pi_i(G) < \pi_i(G + ij)$, then $\pi_j(G) > \pi_j(G+ij)$,*

G - ij is the network resulting from deleting a link ij from the network G and G + ij is the network resulting from adding a link ij to the network G. From this definition, network G is stable if no firm can obtain higher profit from deleting one of its links; and any other link between two firms would benefit only one of them. The definition of the efficiency of a network that is given as follows and is determined by the total welfare generated from that network.

Definition 2 (Network Efficiency) *Network G is said to be efficient if no other network G ′ can be generated by adding or deleting links, such that TW(G ′) > TW(G).*

THE OUTCOMES

Isolated Star Networks

This section focuses on isolated star networks where each star encompassing all firms.[7] The individual equilibrium behavior is rooted in the network structure through their links (equation 1) and the individuals behaviors impact on the aggregate behavior. We examine the possible changes that occur to the individual and aggregate outcomes of the star networks if the density increases by increasing the size of the peripheral firms and the cooperation between them. To examine the impact of the size of the peripherals, we start with the smallest star network that only contains on three firms (S_3), including the center firm. Then, we increase the size of the periphery by adding firms and this forms the star networks S_4, S_5 and S_6 as given in Figure 1.

The impact of the growth of the periphery size on the outcomes varies with the market structure. If firms sell independent goods, the increase of the peripheral firms has positive impacts on the outcomes. In the sense that the R&D investments, the production quantity and the profits of firms, and the total welfare increase with the number of firms in the periphery.

If goods are homogeneous, it seems that the growth of the peripheral firms has both negative and positive impacts on the outcomes. The negative impact appears on the R&D efforts where a new link from the center reduces the individual investment. The another negative impact of growing the peripheral firms appears on the profits of the existing peripherals and this makes the industry profit decline in the size of the peripheral firms. The positive impact is observed on the central firm for small values of the spillover and on the total welfare if the spillover is not high. For the aggregate quantity, the increase of the peripheral firms encourages the production of firms.

Proposition 1 *Assume Cournot competition with n firms participate in R&D where the cooperation forms a star network. If goods are independent, with respect to the size of the new firms in the periphery of the center, the equilibrium outcomes are monotonically increasing.*

Proposition 2 *Assume Cournot competition with n firms participate in R&D where the cooperation forms a star network. If goods are homogeneous, with respect to the size of the new firms in the periphery of the center,*

1. the R&D effort of all firms and the production quantity of the existing peripherals are monotonically decreasing.
2. the production quantity and the profit of the central firm are monotonically decreasing for not small values of the spillover.
3. the total welfare is monotonically increasing for not high values of the spillover.

Example 1 *Consider the star networks given in Figure 1.*

1. Figure 2 shows the impact of increasing the size of the periphery of the center on the R&D effort and profit of the existing firms.
2. Figure 3 shows the impact of increasing the size of the periphery of the center on the aggregate quantity, the industry profit and the total welfare.

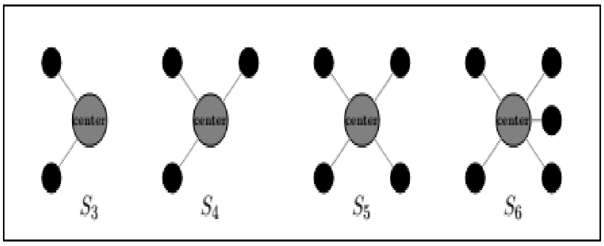


Figure 1 Star networks of sizes from three to six.

The study of the impact of the new peripherals on the centers profit provides two observations. The first observation concerns the change rate of the centers profit with growing the peripheral firms. The steady increase in the activity levels does not generate a constant change in the profit of the centers. Meaning that, the difference between the profits of the centers in S_3 and S_4 is not equal to the difference between the profits of the centers in S_4 and S_5 (see Figure 2). The second observation concerns the stability of the network. Since the profit increases with the links, the peripheral firms seek to cooperate with each other and this means the star network is not stable (see previous example).

The following proposition relates the optimal profit of the central firm to the links between the peripherals. If goods are independent, the cooperation between peripheral firms raises the center's profit. However, if goods are homogeneous, the opposite is observed for each link established among peripheral firms. The following proposition states this point and Example 2 illustrates it.

Proposition 3 *Assume Cournot competition with n firms participate in R&D where the cooperation forms a star network. With respect to the size of the cooperation among firms in the periphery of the center,*

1. *the profit of the central firm is monotonically increasing if goods are independent.*
2. *the profit of the central firm is monotonically decreasing if goods are homogeneous.*

Example 2 *Consider the star network S_6 given in Figure 4 (left). In the networks GS_1, GS_2 and GS_3, some of the peripheral firms are linked. The figure shows the profit of the central firm with respect to the size of the cooperation among firms in the periphery of the center.*

[7] The role of the star networks (centers) has been discussed in many empirical papers (e.g., Jaffe 1986; Stuart 2000; Tomasello et al. 2013)

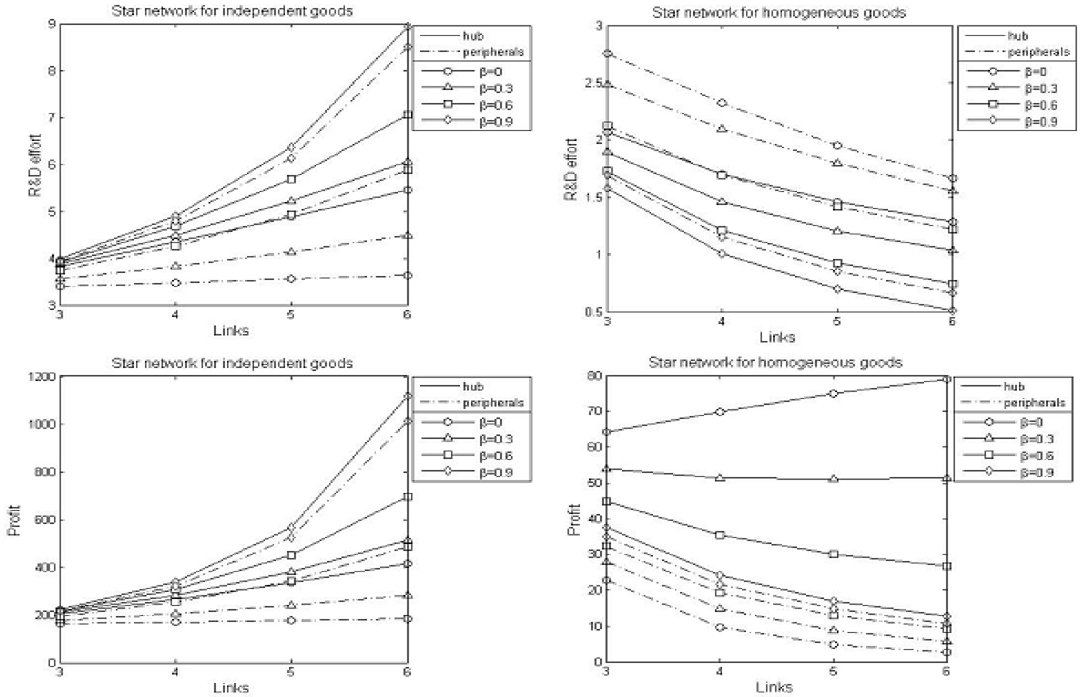


Figure 2 R&D effort and profit with respect to the size of periphery. The graphs in the left side show the outcomes for independent goods and the graphs in the right side show the outcomes for homogeneous goods. The parameters used to plot the results are a = 120, $\bar{c}$ = 100 and γ = 1 (γ = 2) for homogeneous (independent) goods.

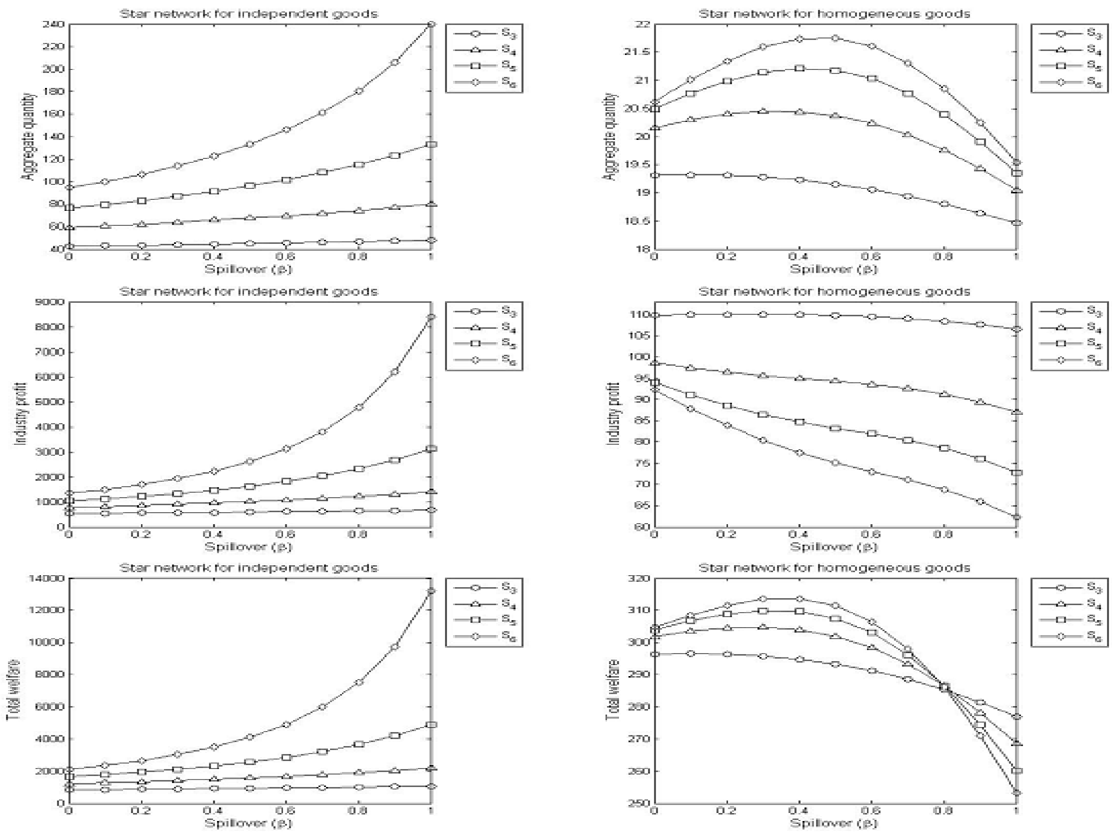
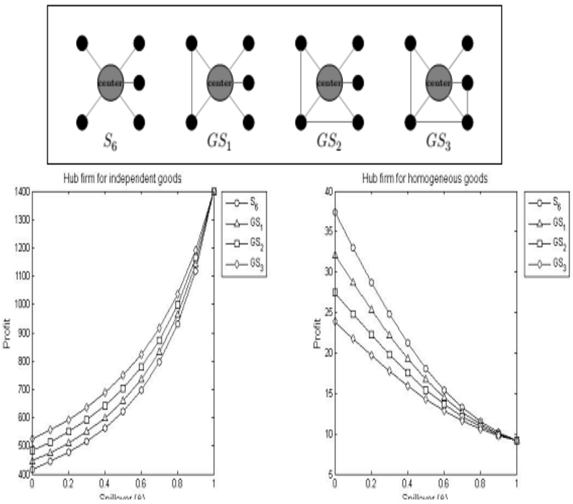


Figure 3 Aggregate quantity, industry profit, and total welfare for the star networks given in Figure 1 .The graphs in the left side show the outcomes for independent goods and the graphs in the right side show the outcomes for homogeneous goods. The parameters used to plot the results are a = 120, $\bar{c}$= 100 and γ = 1 (γ = 2) for homogeneous (independent) goods.



Interlinked Star Networks

The objective of this section is to examine the effect of the connectivity of two star networks via the centers and peripheral firms on the equilibrium outcomes. To obtain some feeling for this, let us examine an example of two connected star networks. We start with two components, each component constitutes a star network with the same number of firms (network G_1 in Figure 5). Then, we join the two centers in the two disconnected stars to form an interlinked star network (network G_2). After that, we join some firms in the periphery in the network G_2 to form the network G_3.

Firstly, the impact of the links between the centers on the equilibrium outcomes. From the center profit standpoint, the highest profit is acquired when the centers are linked (the network G_2), irrespective of the product type.

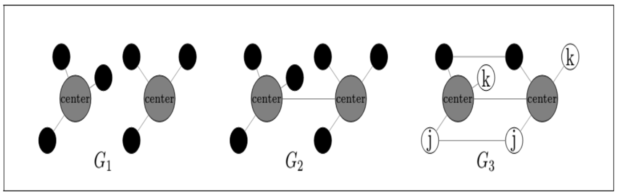


Figure 5 Two disconnected star networks (left), interlinked star networks by their centers (middle) and interlinked star networks by centers and peripheral firms (right). In the networks G_1 and G_2, there are two types of firms the central and the peripheral firms. In the network G_3, there are three types of firms the center, the linked peripheral firms (j) and the peripheral firms (k).

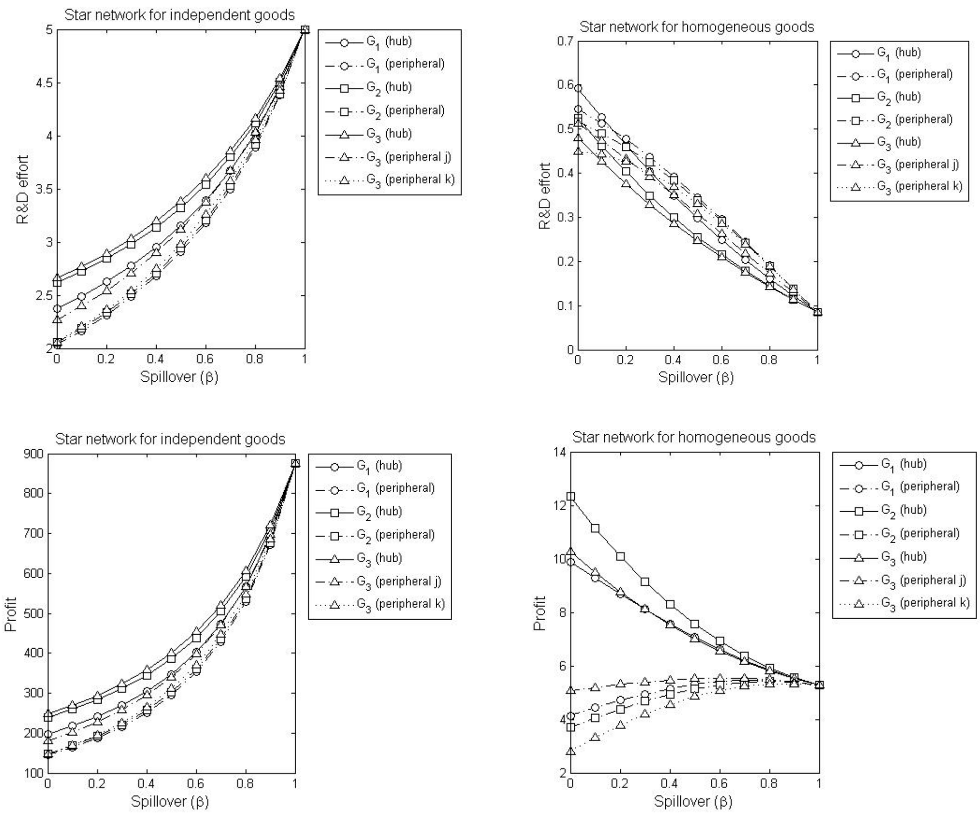


Figure 6 R&D effort and profit of firms in the networks in Figure 5. The parameters used to plot the results are a = 120, $\bar{c}$ = 100 and γ = 2 (γ = 3) for homogeneous (independent) goods.

For other equilibria, when comparing the results of the networks G_1 and G_2, it seems that if goods are independent, the outcomes are improved. However, if goods are homogeneous, the R&D efforts of all firms and the profits of the peripheral firms decrease. Also, firms in the network of disconnected stars seem more productive and the industry profit is higher in that network than in the interlinked stars by centers if the spillover is sufficiently high. Moreover, the disconnected stars seem more efficient than the interlinked centers if the spillover is not small. This indicates that in the case of homogeneous goods, the expected benefit of linking the centers is limited to those central firms since their profits increase.

Secondly, the influence of the links between the peripheral firms on the outcomes. When comparing the outcomes of the networks G_2 and G_3 for independent goods, we find that the links between peripherals are positive factors for the outcomes. For homogeneous goods, we observe the opposite for the R&D effort and the profit of the centers. However, the interlinking of peripheral firms seems a positive factor for the aggregate production and the total welfare if the spillover is small; otherwise they are high if the star networks stay separated.

Proposition 4 *Assume Cournot competition with n firms participate in R&D where the cooperation forms two disjoint star networks. When the two star networks forms one component,*

1. *the equilibrium outcomes increase if goods are independent.*
2. *the profit of the centers increases and the total welfare decreases for most values of the spillover if goods are homogeneous.*

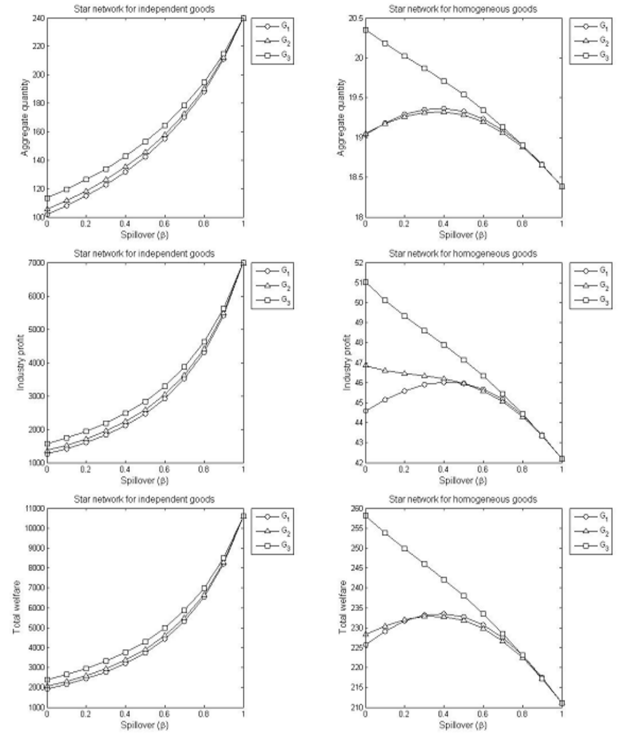


Figure 7 Aggregate quantity, industry profit, and total welfare for the networks in Figure 5. The parameters used to plot the results are $a = 120$, $\bar{c} = 100$ and $\gamma = 2$ ($\gamma = 3$) for homogeneous (independent) goods.

Example 3 *Consider the star networks given in Figure 5. Figures 6 and 7 show the equilibrium outcomes before and after linking the central and peripheral firms.*

CONCLUSION

In this paper, we considered the network game to study the impact of the growth of the star network on the equilibrium outcomes. The analysis of the outcomes showed the significant benefits behind remaining firms in the center of the network. The increase of the profit is the priority of firms that is acquired by forming several R&D relationships, irrespective of the product type. This would allow the central firms to preserve the characteristic features of the R&D network. However, the benefit resulting from growing the size of the periphery depends on the type of the products of firms. If the products are differentiated, the increase of the density leads to better outcomes for all equilibria. If the substitution between goods is high, the increase of the peripheral firms leads to higher aggregate production quantity and total welfare.

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References

Ahuja, G. 2000. Collaboration Networks, Structural Holes, and Innovation: A Longitudinal Study. Administration Science Quarterly., 45: 425-455.

Cantner, U. and Graf, H. 2006. The network of innovators in Jena: An application of social network analysis. Research Policy., 35: 463-480.

D'Aspremont, C. and Jacquemin, A. 1988.Cooperative and Noncooperative R&D in Duopoly with Spillovers. American Economic Review., 78: 1133-1137.

Deroian, F. 2008. Dissemination of spillovers in cost-reducing alliances. Research in Economics., 62: 34-44.

Goyal, S. and Joshi, S. 2003. Networks of collaboration in oligopoly. Games and Economic Behavior., 43 (1): 57-85.

Goyal, S. and Moraga-Gonzalez, J. L. 2001.R&D Networks. Rand Journal of Economics., 32: 686-707.

Galeotti, A. Goyal, S. and Kamphorst, J. 2006. Network formation with heterogeneous players. Games and Economic Behavior., 54: 353372.

Galeotti, A. and Melndez, M. 2004. Exploitation and cooperation in networks, Tinbergen Institute Discussion Paper 04-076/1.

Jackson, M. 2008. Social and Economic Networks. Princeton, NJ: Princeton University Press.

Jackson, M. O. and Wolinsky, A. 1996.A Strategic Model of Social and Economic Networks. *Journal of Economic Theory*, 71: 44-74.

Konig, M. D., Battiston, S., Napoletano, M. and Schweitzer, F. 2012. The efficiency and stability of R&D networks. Games and Economic Behavior., 75: 694-713.

Newman, M. E. J. 2003. The Structure and Function of Complex Networks. SIAM Review., 45: 167-256.

Jaffe, A. B. 1986. Technological Opportunity and Spillovers of R&D: Evidence from Firms' Patents Profits, and Market Value. American Economic Review., 76: 984-1001.

Powell, W. W., White, D. R., Koput, K. W. and Owen-Smith, J. 2005. Network dynamics and field evolution: The growth of interorganizational collaboration in the life sciences. *American Journal of Sociology*., 110(4): 11321205.

Riccaboni, M. and Pammolli, F. 2002.On firm growth in networks. Research Policy., 31(8-9): 14051416.

Song, H. and Vannetelbosch, V. 2007.International R&D Collaboration Networks. The Manchester School., 75(6): 742-766.

Stuart, T. E. 2000. Interorganizational Alliances and the Performance of Firms: A Study of Growth and Innovation Rates in a High-Technology Industry. *Strategic Management Journal*., 21: 791-811.

Tomasello, M. V., Napoletano, M., Garasz, A. and Schweitzer, F. 2013. The Rise and Fall of R&D Networks. arXiv:1304.3623v1.

Verspagen, B. and Duysters, G. 2004. The small worlds of strategic technology alliances.Technovation., 24: 563-571.

Watts, D. J. and Strogatz, S. H. 1998.Collective dynamics of 'small-world' networks. Nature., 393: 440-442.

Westbrock, B. 2010. Natural concentration in industrial research collaboration. *RAND Journal of Economics*., 41 (2): 351-371.

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